How to Sell a (Bankrupt) Company*

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Abstract. The restructuring of a bankrupt company often entails the sale of such company. This paper suggests a way to sell the company that maximizes the creditors' proceeds. The key to this proposal is the option left to the creditors to retain a fraction of the shares of the company. Indeed, by retaining the minority stake, creditors reduce to a minimum the rents that the sale of the company leaves in the hands of the buyer.

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1. Introduction

A bankruptcy procedure — or, even before bankruptcy, any restructuring in a situation of financial distress — has to choose the destiny of the insolvent firm. Usually the ownership and control of the company is transferred in new hands, which are in general different from the previous owners or even from the creditors (who have the control during the bankruptcy procedure). In other words, bankruptcy often leads to the sale of the company. This paper suggests a way to sell a bankrupt company that maximizes the creditors proceeds from the sale.

Maximizing the creditors' proceeds from the sale of a bankrupt company is not the first quality of a bankruptcy procedure that comes to mind. Indeed, a bankruptcy procedure is usually considered efficient if it allocates the company assets in the hands of individuals that maximize the value of the company. We label this quality of a bankruptcy procedure *ex-post efficiency*.

Ex-post efficiency does not take into account the effect that the destiny of the bankrupt company has on the incentives of the involved parties before the firm goes into bankruptcy, even before any clue of financial distress is at the horizon. A bankruptcy procedure that does a good job at promoting these incentives can be regarded as *ex-ante efficient*.

Two groups of stake-holders play a critical role in the life of a company. These are the entrepreneurs or managers of the company and its creditors. A bankruptcy procedure 'punishing' managers or entrepreneurs of the insolvent firm (for example not giving them control even when it is ex-post efficient to do so) may be seen as ex-ante efficient. It provides entrepreneurs with the right incentives to manage the firm so as to avoid ending up in financial distress, for example by not undertaking too many risks. The effects of different bankruptcy procedures on the managers' and entrepreneurs' incentives have been extensively studied in the literature (e.g. Aghion and Bolton 1992, Berkovitch, Israel, and Zender 1993, Bolton and Scharfstein 1996).

This paper focuses on a different aspect of ex-ante efficiency: the protection of the creditors' claims. By protection of creditors' claims we mean the attempt to maximize

the proceeds to the creditors from the reorganization of the firm. The revenues to the creditors may seem, from an ex-post point of view, a pure transfer and therefore irrelevant. However, a bankruptcy procedure which maximizes creditors' proceeds from the sale of the company when it is in financial distress may reduce the company's overall costs of borrowing. This has clear efficiency implications. Investment projects that would be financed under a bankruptcy procedure which protects creditors' claims would not be financed under bankruptcy procedures which allocate the company efficiently but sacrifice creditors' revenues.[1]

Key to our proposed way to sell a bankrupt company is a very simple point: it is never optimal to sell the entire ownership of the company. Instead, it is always optimal to leave the creditors the option to retain an equity stake in the distressed firm. Indeed, it is possible to transfer the control of the company in the hands of the individual that maximizes its value without transferring all the shares in his hands. Hence, by retaining a minority stake in the company creditors can capture the entire increase in the market value of the company at least on this minority stake and in so doing maximize their returns.

Of course, if creditors knew the value of the company in the hands of potential buyers then maximizing revenues would be even easier. They could make a take-it-or-leave-it offer to the buyer who is willing to pay more and capture all the increase in value of the firm. However, one of the major sources of complexity and delays in bankruptcy is the difficulty in evaluating what will be the value of the company in different hands.[2] Potential buyers value the company differently because they may have different plans for the future or because of synergies with their other businesses. Creditors will in general not know for sure how much these buyers are prepared to

[1]The observation that protecting creditors' claims has clear efficiency implications may seem surprising, given that it is usually argued that giving creditors too much power in a bankruptcy procedure may induce them to liquidate too often (e.g. Aghion, Hart, and Moore 1992, Franks and Torous 1989). However, this happens when creditors, by liquidating, can be entirely reimbursed. Clearly in this case increasing revenues is not a creditors' concern. However, if — as usual in a bankruptcy situation — the value of the company, even when maximized, is less than the sum of the credits, creditors will want to maximize their revenues.

[2]See, for example, the cases of Sunbeam-Oster (HBS # 5-293-046) and Marvel Entertainment Group (HBS # 5-298-028).

pay and will need to rely on the competition among buyers to identify the individual who is willing to pay more for the company. However, if the company has different values in the hands of different individuals, competition among buyers is not perfect and creditors will not be able to capture the whole value of the company. As a result, the buyer is able to obtain the company for a price lower than its value. In many situations, the value attached to a bankrupt company may differ so much among potential buyers that the price may end up to be substantially lower.

Our proposal aims to reduce this rent which is left to the buyer, increasing the returns to the creditors who are selling the company. The intuition is very simple: by transferring control and retaining an equity stake in the company, the creditors can make sure that at least on this equity stake they capture the full value of the company and minimize the rents left in the hands of the efficient buyer. In other words, by auctioning off only a fraction of the company, the creditors reduce the differences among potential buyers, making in this way competition stronger and therefore reducing the buyer's rents.

We show that the optimal way to sell the company is to auction off a fraction of its equity (but always a fraction which entails control) and identify the size of this fraction in different situations. In particular, when control does not entail any private benefits we show that it is always optimal to sell only the minimum stake necessary to transfer control (Section 3). In other words, it is optimal to separate completely the voting rights from the cash flow rights of the company: the creditor should sell all the voting rights and possibly retain all the cash flow rights. This is due to the fact that in the absence of private benefits from control the individual who is willing to pay more for the company is also the efficient buyer (i.e. the one who maximizes the value of the company ex post).

However, one might argue that when there are no private benefits from control buyers should not have different willingnesses to pay. Even if the company has higher value in someone else's hands, an individual can always acquire the control and then resell it to someone who values it more. If, when bidding, the potential buyers take into account the additional revenues from reselling the control stake, the amount each

buyer is willing to pay contains a common component, due to the option to resell. We show that even in this case it is still optimal to auction off only the minimum control stake of the company (Section 4). In fact, when reselling the company, a seller will be able to capture only part of the value the company in the hands of the buyer. Therefore the value of the option to resell in general does not reflect the full increase in the value of the company due to the transfer of control. By retaining a minority stake instead the creditors can guarantee themselves the whole increase in the company's value on this stake.

When the control of the firm in distress entails some private benefits, it is no longer optimal to sell only the minimum control stake. Private benefits of control, in fact, create a trade off between ex post and ex ante efficiency, since the bidder who is willing to pay the most for the minimum control stake of the company might not be the one who maximizes the company's value. However, it might still be optimal for the creditors to retain part of the equity stake of the firm (Section 5), but not necessarily the minimum stake necessary to transfer control. In other words the creditors do not want to separate completely the voting rights from the cash flow rights of the company. Bundling these rights together but retaining as much as possible of the cash flow rights of the firm allows the creditors to maximize their returns and to attract the most buyer in whose hands the company's value is highest. The optimal mechanism is then an auction of the lowest control stake that renders this buyer also the individual with the highest willingness to pay for the company. In so doing the creditors maximize the price paid by the buyer for the control stake of the company (the voting rights) and, at the same time, the value of the minority stake (the cash flow rights) left in their hands.

In most of our analysis the choice of the selling procedure which maximizes the creditors' revenues does not imply a trade-off between ex-post and ex-ante efficiency. Indeed, the mechanism which we derive as optimal also allocates the company in the hands of those who maximize its value (and in the case of private benefits we adjust the fraction sold so that this result is still true). However, creditors have also the option to further increase their proceeds by introducing a reservation price. This

introduces a trade-off between ex-ante and ex-post efficiency, since a reservation price entails a loss in ex post efficiency. We show that reducing the fraction of the equity auctioned off reduces the ex-post inefficiency associated with the reservation price. In other words, when the seller uses a reservation price, reducing the control stake auctioned off improves both ex-post and ex-ante efficiency.

A question that comes to mind, given the results described above, is whether this bankruptcy procedure could be implemented in a decentralized way. In other words, whether it is possible to transform the firm in distress in an all equity company, distribute the shares of this company to the creditors and leave them free to decide the fate of this new all-equity company. This would be equivalent to a privatization of the bankruptcy procedure. In Section 6 we show that this procedure may achieve the same revenues obtained by the centralized procedure (i.e. the optimal selling procedure discussed above). However, this is only one of a whole set of equilibria of the creditors' tendering game. Some of the equilibria of this game may be inefficient and prevent the creditors from maximizing their returns since each creditor may have an incentive to free-ride on other creditors when deciding whether to transfer the control of the company in the hands of the efficient buyer. In other words, a bankruptcy law that disciplines and centralizes the creditors behaviour in bankruptcy may be preferred to privatizing the bankruptcy procedure.

The main result of our analysis can shed light on some of the features of observed bankruptcy cases. Usually, an observed increase in the creditors' equity stake at the end of a bankruptcy restructuring is explained by the need to increase monitoring by large shareholders (see for example Gilson (1990)), or more generally by the fact that an increase in the creditors' stake might affect the value of the company. This paper suggests that this might simply be the best way for the creditors to sell the firm and recuperate as much as possible of their credits.

The analysis of this paper is relevant not only for the change of control in a bankruptcy procedure, but also for any transfer of control. The reason we are focusing on bankruptcy is that this is a natural environment in which a party (the creditors) is the owner of a company and would rather sell for the highest possible return.

Whenever the transfer of control takes place in a non decentralized way, our result still applies. An interesting other case to which our result applies is the spinoff of a division of the company. In this case we show that the selling party (the original company) has an interest in retaining an equity stake in the spinoff company. Therefore, the IPO should be done only for a fraction of the equity and the remaining shares should be sold in the market afterwards. In the next section we relate our paper to other papers that focus on the transfer of control, and show how our results apply in that context.

The rest of the paper is structured in the following way. We review the related literature in Section 2. Section 3 presents the main result of the paper in the absence of any private benefit from control and under the assumption that potential buyers cannot trade among themselves their acquired stake in the company. In Section 4 we prove that the same result holds when we remove the latter assumption. We then analyze in Section 5 how the result generalizes to the case in which the control of the company entails private benefits. Section 6 suggests how to implement the optimal selling procedure of the bankrupt company and analyzes the possibility of privatizing it. Section 7 concludes.

2. Related Literature

The literature on bankruptcy is vast. However, very little of it is focused on how to sell a bankrupt company and in general on the protection of creditors' claims. This is the reason why the papers most closely related to ours are concerned mainly with the transfer of control rather than with bankruptcy (Zingales 1995, Bebchuk 1994). This is consistent with our claims that the results of our analysis are relevant for any transfer of control even outside a bankruptcy procedure.

Zingales (1995) is the closest paper to ours. It analyzes how the owner of a firm can extract the highest possible surplus from a raider. Zingales shows that the incumbent may want to sell the minority stake of the firm on the stock market before facing the raider, in order to free-ride on any increase in the value of the firm induced by the transfer of control. The main difference with our analysis lies in the fact that

Zingales focuses on the case in which only one raider is planning to take over the firm, while we consider the case where there is competition among potential buyers for the company.

In Zingales (1995), the incumbent, if he owns the entire company when bargaining with a unique potential buyer, will not be able to extract any additional surplus from the raider by selling only the control stake of the firm. In fact, when the incumbent bargains with the raider, the reservation price that makes him indifferent between selling or not the firm will adjust. As a result, the amount of surplus the incumbent will be able to extract is the same whatever stake of the company is sold. However, this is not true if the incumbent has transformed the minority stake of the firm in cash in advance by selling it on the stock market. Therefore, in Zingales (1995) the only way in which the incumbent will be able to maximize the rent he extracts from the raider, even in the absence of private benefits from control, is by selling the minority stake of the firm on the stock market in advance.

In our analysis, this is not true. Indeed the presence of competition among potential buyers for the firm prevents the reservation value of the incumbent (the creditors in our case) from adjusting when selling only the control stake. Therefore it is strictly optimal for the creditors to retain the minority stake of the firm so as to extract the highest surplus from the potential buyers.[3]

The other paper on the transfer of control that is relevant for our analysis is Bebchuk (1994). This paper analyzes the efficiency properties of different procedures for the sale of control of a company in the presence of private benefits from control. Bebchuk shows that a procedure that does not give any say to the minority shareholders of the company (market rule) may result in inefficient transfers of control, while a procedure that does give a veto power to minority shareholders (equal opportunity rule) may prevent efficient transfers of control. The paper is closely related to the analysis we present in Section 5.

[3]Also in the case in which there is only one potential buyers, if the incumbent does not know the buyer's willingness to pay, our result holds, and it is optimal to use the number of shares sold as a screening device (Cornelli and Li 1997).

In Bebchuk (1994) the critical condition that yields (ex post) inefficiencies in the transfer of control is whether the private benefits of the seller and the buyer of the company are positive or negatively correlated with the benefits that are shared by the minority shareholders. The equivalent condition in our analysis (Section 5 below) is whether the private benefits of potential buyers are positively or negatively correlated with the public or transferable benefits associated with their shareholding. The main difference with our analysis is that, since we consider a structured procedure, creditors with minority stake will not free-ride, hence the transfer of control will always be ex-post efficient. However, the correlation between private and public benefits will determine the proportion of shares in excess of the minimum necessary to transfer the control that creditors will decide to auction off. In a privatized bankruptcy procedure, however, creditors have an incentive to free-ride and ex-post inefficiencies may arise (Section 6).

Another paper of relevance for our analysis is Riley (1988). This paper shows that in the sale, for example, of oilfields the expected revenue of the seller is raised by using royalty rates. In other words the seller increases its revenues by making the winner's payment a function of the information revealed during the auction and of any signal of the value of the object auctioned off that might become available after the oilfield is sold. The relationship with our analysis can be seen in the similarity between royalty rates and cash flow rights. However, Riley's result holds only when the values of the oilfield in the hands of the potential buyers are correlated across buyers (the case analyzed is affiliated values) while our result holds also when the firm's values in the hands of potential bidders are independent (see Section 3).

In particular, in Riley (1988) royalty fees allow the price paid by the winning bidder to depend on the entire information on the value of the oilfield revealed during the auction as well as on any information revealed after the auction. Whenever the information revealed does not affect the values of the oilfield to potential buyers, royalty fees do not affect the seller's revenue.

Our result instead holds also when the information revealed in the auction does not affect the different values of the firm in the hands of potential buyers. Indeed, our

result depends on the fact that it is possible to transfer the control of a firm without necessarily transferring all the cash flow rights.

Finally, few recent papers have discussed the role of auctions in bankruptcy. Baird (1986) and Aghion, Hart, and Moore (1992) argue that in a world without cash or credit constraints (like the one we are analyzing) auctions are an efficient bankruptcy procedure, distributional issues not withstanding. We do not disagree with this point. However, we argue that an auction achieves ex post efficiency (since it allocates the firm's control optimally) but does not necessarily maximize the creditors' proceeds, if the creditors are required to auction off the entire company, as it usually happens in bankruptcy procedures. In other words, modifying the procedure so as to allow the creditors to auction off only the control stake of the firm may increase creditors' revenues. Notice that the fact that it is optimal for the creditors to retain an equity stake in the company has the flavor of non-cash auctions (as in Aghion, Hart, and Moore (1992) and Rhodes-Kropf and Vishnathan (2000)), where bidders may offer to the seller equity stakes in the company. However, we show below that in our set-up it is never optimal for the bidders to spontaneously offer equity stakes (since it reduces their rent) and therefore it is up to the sellers (the creditors) to obtain it by reducing the control stake sold.

3. How to Sell the Company

Let us consider a firm, whose capital structure consists of common stock and straight debt, which has declared bankruptcy. The debt is owned by N creditors.

Creditors may be compensated with cash and with share participation in the re-organized firm. We rule out the possibility to compensate creditors through debt claims in the re-organized firm. In what follows we show that this implies no loss in generality.

How the creditors share the returns from the re-organization of the firm is not relevant for our analysis: our result holds true whatever way the creditors choose to share the returns. The only thing that is relevant from our view point is the sum of the returns to all creditors.

We characterize the optimal way to sell this company. Assume that the value of the firm depends on who acquires the control stake of the firm. In particular we take the company to have different values depending on who obtains the control. Let us denote the value of the firm in the hands of individual i as V_i. We further assume that an individual does not need to acquire all the shares of a firm to have the control. In particular we take $0 < \underline{\alpha} < 1$ to denote the amount of shares necessary to have the control of the firm.[4]

In this section we assume that these values V_i are specific to each potential buyer and are independent across them (private values). The next section however considers the case in which whoever obtains the control of the firm can resell it to someone who could increase the company value. If in this way the original buyer could increase his payoff, the resulting situation would be one of common rather than private values. Finally, in Section 5 we analyze the case in which the control of the firm entails private benefits from control.

All three cases are analyzed in two scenarios. First we consider the full information case with two potential buyers and assume that the mechanism to allocate control is an auction. All the intuition of the results can be obtained from the full information case. However, one may object that when the seller knows perfectly well what is the buyers' willingness to pay, he does not need to set up an auction: he will just make a take-it-or-leave-it offer to the buyer with the highest willingness to pay and extract all the surplus from the buyer. Of course, this is not a realistic situation: creditors do not know what will be the firm's value in the hands of other investors. Therefore, we also develop a general model with asymmetric information where we prove that the auction is optimal. This is done in order to make sure that our recommendation (not to sell the entire company) does hold in the realistic situation in which creditors do not know the potential buyers' willingness to pay.

One may argue that—although auctions have been recommended as the best method to sell the company in a bankruptcy procedure—in reality other methods

[4]We take $\underline{\alpha}$ to be exogenous in the paper, we discuss in the conclusions what is the optimal level of $\underline{\alpha}$ if the creditors are free to choose the control stake of the bankrupt firm.

are used (for example, Chapter 11 is a bargaining procedure). In the context of our paper the auction is only one of the optimal selling procedures which can be used. Other indirect mechanisms will implement the optimum. We use an auction only because it is easier to convey the intuition in that context. What is important is that any optimal mechanism will involve the sale only of a control stake of the company.

3.1. The Perfect Information Case

Consider a situation in which there exist only two potential buyers, labelled 1 and 2, for the insolvent firm, none of them a creditor.[5] Each potential buyer has a specific plan on how to run the company if in control and the firm, under his control, has value V_1 and V_2, respectively. Without loss of generality, let us assume that $V_1 < V_2$. We assume that the entire valuation V_i, $i = 1, 2$, represents the firm's market value, transferable and public, and the control of the firm does not yield any private benefit. We analyze the case with private benefits in Section 5.

We show that in this situation it is never optimal for the creditors to sell the entire company. If the creditors sell the entire company through an auction, the unique equilibrium of the auction is such that buyer 2 obtains the firm at the price V_1.[6] This is ex post efficient, since the value of the firm is maximized in the hands of buyer 2. However, the creditors could have obtained a higher revenue by structuring the auction differently.

Assume instead that only the minimum number of shares necessary to have control, $\underline{\alpha}$, is auctioned off.[7] Then buyer 2 buys $\underline{\alpha}$ shares and obtains the control, paying

[5]This assumption is needed to simplify the analysis of the equilibrium outcome of the auction. Indeed, in the event that a potential buyer is one of the creditors there would exist incentives for him to overbid as exemplified in Burkart (1995) and Bulow, Huang, and Klemperer (1999). The result presented below, however, still holds.

[6]Notice that the equilibrium described is the unique trembling-hand-perfect equilibrium of this simple auction game. Here trembling-hand perfection is used in a standard way to prevent bidder 1 from submitting a bid (not selected in equilibrium) that exceeds the value the firm has in his hands. Notice also that this result holds true when the auction is structured as a first price auction.

[7]We discuss in the conclusions the case in which $\underline{\alpha}$ is endogenized and the creditors can choose the voting structure of the control shares.

$\underline{\alpha}V_1$. The creditors are now left with a minority stake $(1-\underline{\alpha})$ of a firm whose total value is V_2. The total revenue accruing to the creditors are:

$$\underline{\alpha}V_1 + (1-\underline{\alpha})V_2 > V_1. \tag{1}$$

Notice that, unless the creditors decide to auction off only the control stake of the firm, the competition between the two buyers never leads to the equilibrium bid $[\underline{\alpha}V_1 + (1-\underline{\alpha})V_2]$. In other words, the buyers never voluntarily bid for only a fraction of the firm, since bidding for the entire firm maximizes the surplus appropriated by the winner, $(V_2 - V_1)$.

Of course, another way to obtain the same revenues is to auction off the entire firm with a reservation price of $\underline{\alpha}V_1 + (1-\underline{\alpha})V_2$. The possibility to auction off only the control stake of the firm is then useful to identify the highest credible reservation price. However, in a perfect information setting it is not meaningful to talk about reservation price (since the seller knows the buyers' willingness to pay), so we will discuss reservation prices only in a setting of asymmetric information, where we can look for the optimal way to sell the company (instead of assuming that the company is sold through an auction).

3.2. The Private Information Case

Let us now assume that each valuation V_i is private information of buyer i but it is common knowledge that each V_i is drawn independently from the same distribution function $F(\cdot)$ over the interval $[0, \bar{V}]$, with density $f(\cdot)$. If $V = (V_j)_{j\in N}$, and $V_{-i} = (V_j)_{j\in N, j\neq i}$, we can define

$$G(V) \equiv [F(V_j)]^N$$

and

$$G_{-i}(V_{-i}) \equiv [F(V_j)]^{N-1}$$

with corresponding densities $g(V)$ and $g_{-i}(V_{-i})$.

Let us look at the selling procedure which maximizes the creditors' revenue.

By the Revelation Principle, it is possible to restrict attention to the direct revelation mechanisms where the buyers simultaneously announce their valuation $\tilde{V}_i$ to the creditors and the creditors choose the mechanism $\{p_i(\tilde{V}), t_i(\tilde{V}), \alpha\}$, where $p_i(\tilde{V})$ is the probability that buyer i gets control; $t_i(\tilde{V})$ is the amount he has to pay and α is the proportion of shares sold. We look for a Bayesian Nash equilibrium of this mechanism in which buyers truthfully reveal their own valuations.

If the firm has value V_i under the control of buyer i, then his expected payoff when declaring $\tilde{V}_i$ is given by the value of his equity stake minus the payment to creditors:

$$U_i(V_i, \tilde{V}_i) \equiv \int_{V_{-i}} \Big\{ \alpha V_i \, p_i(\tilde{V}_i, V_{-i}) - t_i(\tilde{V}_i, V_{-i}) \Big\} g_{-i}(V_{-i}) dV_{-i}. \tag{2}$$

The creditors revenues are given by the total payments from the buyers plus the expected value of the minority stake remaining in their hands:

$$\int_V \left[\sum_i t_i(V) + \sum_i [1-\alpha] V_i \, p_i(V) \right] g(V) dV. \tag{3}$$

The creditors maximize their revenues in (3) with respect to α, p_i and t_i subject to several constraints. The individual rationality constraint (which guarantees that each buyer is willing to participate):

$$U_i(V_i, V_i) \geq 0, \ \forall i \in N, \ \forall V_i \in [0, \bar{V}], \tag{4}$$

the incentive compatibility constraint (which guarantees that each buyer will declare his true value V_i)

$$U_i(V_i, V_i) \geq U_i(V_i, \tilde{V}_i), \ \forall \tilde{V}_i \in [0, \bar{V}], \ \forall i \in N, \ \forall V_i \in [0, \bar{V}], \tag{5}$$

and

$$\sum_i p_i(V) \leq 1, \tag{6}$$

$$\underline{\alpha} \leq \alpha \leq 1. \tag{7}$$

The incentive compatibility condition, constraint (5), can be rewritten as a maximization problem. The first and second order conditions of such problem are then necessary to guarantee that truth telling is optimal for all the bidders. Following Myerson (1981), we show in Appendix A.1 how we can utilize the first order conditions of (5) to transform the objective function of the creditors (3) into the following:

$$\int_V \left\{ \sum_i \left[V_i - \alpha \frac{1 - F(V_i)}{f(V_i)} \right] p_i(V) \right\} g(V) dV \tag{8}$$

We can now derive what is the best way in which creditors should sell the company.

Proposition 1. *If $F(V)$ has a monotonic increasing hazard rate, the optimal selling procedure is an auction where the creditors sell $\underline{\alpha}$ shares to the highest bidder.*

Proof: The objective function (8) is decreasing in α, therefore it is optimal to set α as low as possible. Once we set $\alpha = \underline{\alpha}$ the problem coincides with Myerson (1981)'s optimal auction problem. Hence the optimal selling procedure is an auction. Further, by looking at the second order conditions of the incentive compatibility problem (5) derived in Appendix A.1 it is easy to see that they are satisfied for a constant $\alpha = \underline{\alpha}$. ■

Therefore, also in a general set-up it is always optimal to sell the minimum possible number of shares, $\underline{\alpha}$.

Notice that the above selling mechanism is ex-post efficient, since the firm is allocated in the hands of the investor who maximizes its value. However, this is due to the fact that we ignored the possibility to impose a reservation price. In the corollary below we introduce this possibility.

Corollary 1. *It is optimal for the creditors to sell the company to buyer i only if*

$V_i \geq V^*$, where V^* is defined so that

$$V^* - \underline{\alpha}\frac{1 - F(V^*)}{f(V^*)} = 0$$

.

Proof: It is easy to see that if $V_i < V^*$ then $V^* - \underline{\alpha}\frac{1-F(V^*)}{f(V^*)} < 0$ and it is therefore optimal to set $p_i(V) = 0$. ■

The reservation price introduces a trade-off between ex ante and ex post efficiency. Setting a reservation price increases the creditors' expected revenues, but it introduces some ex post inefficiency. This inefficiency arises when the buyer with the highest willingness to pay has a valuation V_i lower than V^* (or, in terms of the auction, his bid is below the reservation price). In this case the firm will not be sold, although its value is maximized in the hands of that buyer.[8]

An important observation, however, is that the inefficiency introduced by imposing a reservation price is reduced if we do not sell the entire company. In fact, if we sell a fraction α of the company, V^* is given by $V^* - \alpha[1 - F(V^*)]/f(V^*) = 0$. Since we are assuming that $F(V_i)$ has a monotonic increasing hazard rate $h(V_i) = f(V_i)/[1-F(V_i)]$, V^* decreases if α decreases:

$$\frac{\partial V^*}{\partial \alpha} = \frac{h(V^*)}{1 + \alpha\left(\frac{d\,h(V^*)}{d\,V^*}\right)} > 0.$$

In other words, if creditors sell a lower fraction of the company, the reservation price also decreases and, consequently, the ex-post inefficiency introduced by the reservation price is reduced. Therefore, reducing the fraction of equity sold increases both ex ante and ex post efficiency.

[8] We are assuming that the firms has no value if it remains in the hands of the creditors. It is possible to assume that the firm has a value also in the hands of creditors and this introduces an additional reason for introducing a reservation price (that does not increase ex post inefficiency). All the results of the paper will hold.

4. Trading among bidders

One possible objection to the procedure suggested above is that the result relies on the fact that we do not allow the buyers to trade the (control stake of the) firm, once it is in their hands. One might argue that if we allow the buyers to trade stakes of the firm between themselves the value of the firm would be the same for all the bidders. Therefore selling a control stake would be equivalent to selling the entire firm.

In this section we show that our result holds even if we allow buyers to trade stakes of the firm among themselves. In other words, it is still optimal for the creditors to retain the minority stake of the firm and to sell only the control stake. The intuition is that, when reselling the company, a bidder will be able to capture only part of the value of the company in the hands of the buyer depending on his bargaining power. Therefore the value of the option to resell in general does not reflect the full increase in the value of the company due to the transfer of control. However, by retaining a minority stake the creditors can guarantee themselves the full increase in value of the company at least on the minority stake they retain.

Once again we proceed in two stages. We first prove the result in the simple two buyers perfect information case and then we generalize it to the case of N buyers with imperfect and asymmetric information.

4.1. The Perfect Information Case with Trading

Consider the case in which we allow trading of the stakes of the firm among buyers. In other words, assume that buyer 1, after purchasing the firm, can resell it to buyer 2. Let trading be organized in the following two periods. In the first period, the creditors of the bankrupt firm auction off either the entire firm or its control stake; while in the second period, buyers may re-trade it between each other.

We start from the second period in which buyers trade between each other. Independently from the number of bidders that participate in the auction, this stage takes the form of a bilateral trade between the bidder who got the firm in the first period (say bidder 1) and the bidder that can maximize the ex-post value of the firm

(bidder 2) — as long as these two bidders are not the same individual, of course. In the second period we can therefore refer to these two players as the buyer and the seller.

If in the first period the entire firm is auctioned off, in the second period it is a weakly optimal strategy for the seller to trade only the control stake of the firm $\underline{\alpha}$ and retain the minority stake for herself (since the same intuition that we derived in the section before holds also here). As a consequence, if the entire firm has been auctioned off in the first period, in the second one we can restrict attention to the case in which the investor who won the auction is going to sell only a fraction $\underline{\alpha}$ of its equity.

To keep the model of bilateral trade as simple as possible we make the standard assumption that with probability ψ the seller (bidder 1) makes a take-it-or-leave-it offer to the buyer (bidder 2), and with the complementary probability $(1-\psi)$ the buyer makes a take-it-or-leave-it offer to the seller.

In order to solve the game, we have to determine the reservation price of both parties in period 2. The highest price the buyer is willing to pay for the control stake is $\underline{\alpha}V_2$ (i.e. his entire surplus from obtaining the control stake $\underline{\alpha}$). The lowest price the seller is willing to accept for the control stake of the firm is slightly more complex. It is the price that makes him indifferent between selling the control stake of the firm or retaining it for himself. If only the control stake of the firm is auctioned off in period one, then this reservation price is $\underline{\alpha}V_1$. If instead the entire firm is auctioned off in period one, then the price for the control stake of the company $\underline{\alpha}V$ is such that $\underline{\alpha}V + (1-\underline{\alpha})V_2 = V_1$.[9]

Consider first the case in which the entire firm is auctioned off in period one. The price the seller is able to obtain in period two for the control stake of the firm is:

$$\underline{\alpha}\left[\psi V_2 + (1-\psi)V\right] \tag{9}$$

[9]For simplicity we assume that $V_1 > (1-\underline{\alpha})V_2$. The whole analysis can be easily adjusted to account for the case in which the above inequality is not satisfied.

which yields a total revenue to the seller equal to:

$$\Pi^* = (1 - \underline{\alpha})V_2 + \underline{\alpha}\,[\psi V_2 + (1 - \psi)V] = \psi V_2 + (1 - \psi)V_1. \tag{10}$$

Equation (10) identifies the highest willingness to pay of bidder 1 in the auction in period one and, hence, the equilibrium winning bid. In other words, equation (10) specifies the total returns to the creditors when they auction off the entire firm in period one.[10]

Consider now the case in which the creditors auction off only the control stake of the firm in period one. The price the seller is able to obtain in period two is:

$$\underline{\alpha}\,[\psi V_2 + (1 - \psi)V_1] \tag{11}$$

This will be the equilibrium winning bid in the auction of the control stake in period one. Hence, the total returns to the creditors are:

$$\Pi^{**} = (1 - \underline{\alpha})V_2 + \underline{\alpha}\,[\psi V_2 + (1 - \psi)V_1] \tag{12}$$

Clearly the returns to the creditors are greater when only the control stake of the firm is auctioned off in period one ($\Pi^{**} > \Pi^*$).

The intuition behind this result is simple. By auctioning off only a control stake of the firm the creditors can guarantee themselves a share of the future value of the firm $(1 - \underline{\alpha})V_2$ that is not going to be affected by the future trade (hence, the bargaining power) between bidders.

A separate issue concerns the case in which the bidder with the higher valuation for the firm is not present at the auction but is available only later on. This is not so unusual in the cases of bankruptcy of large firms, where it is not easy to find immediately the best possible buyers. Sometimes delays in Chapter 11 have been

[10]Equation (10) shows that it does not matter whether bidder 1 trades the entire firm or only its control stake in period two. He is in fact indifferent. The reason is that the reservation value in the bargaining between the seller and the buyer of the firm at time 2 differs in these two cases so as to leave the seller with exactly the same surplus.

justified by the need to look around for the best buyer. We therefore ask whether it may be optimal for the creditors to hold on to the company, waiting for the individual in whose hands the value of the firm is highest to materialize. We show that, even with no discounting, creditors are strictly better off by allocating the control stake of the firm immediately. The reason is that the bidders are able to internalize the possibility to resell the firm and at the auction stage the competition among potential buyers provides the seller with the opportunity to extract a higher surplus from them.

Assume that after the auction an individual, labelled 3, with valuation $V_3 > V_2$ will want to buy the firm and assume no discounting. Assume that this information is known to all the parties to the bankruptcy. If the creditors have not yet sold the firm when buyer 3 appears they can bargain with this buyer and their proceeds are:

$$\psi V_3 + (1 - \psi)\underline{V} \tag{13}$$

where $\underline{V}$ is the value of the firm when kept in the hands of the creditors. As in (12), it does not matter in this bargaining whether the creditors sell the entire firm to buyer 3 or only the control stake.

Assume instead that the creditors auction off the control stake of the firm in period 1 to bidders 1 and 2 and let the winner of this auction bargain with buyer 3 later on. Then the value bidder $i = 1, 2$ expects from the firm is

$$\psi V_3 + (1 - \psi)V_i \tag{14}$$

The winning bid is then $[\psi V_3 + (1 - \psi)V_1]$ and the revenues from the auction are:

$$(1 - \underline{\alpha})V_3 + \underline{\alpha}[\psi V_3 + (1 - \psi)V_1] \tag{15}$$

Notice that even if $V_1 = \underline{V}$ the revenues in (15) are higher than the revenues in (13).

4.2. The Private Information Case with Trading

We now proceed to consider the case in which potential buyers have private information about the value of the firm under their control. To simplify the analysis, we assume that after the shares are sold all V_is are common knowledge. In other words, there is imperfect information only during the sale of the firm. This is admittedly a strong assumption, but it allows us to focus on the issue of revelation of information when creditors sell the firm, which is really what the paper is about, and avoid issues of multiplicity of equilibria that would arise if there were asymmetric information at the bargaining stage.

Assume that creditors have sold α shares to a buyer i with valuation V_i. This value could be the highest possible for the firm or there may exist an individual j whose valuation is higher than V_i. Consider the second case ($V_i < V_j$). As in the previous section, individual i will sell only the minimum control stake to buyer j. The price individual i is able to obtain from a buyer j is

$$\underline{\alpha}\,[\psi V_j + (1-\psi)V]$$

where the lowest price i is willing to accept for the sale of the control stake of the firm $\underline{\alpha}V$ is now

$$\underline{\alpha}V = V_i - (\alpha - \underline{\alpha})V_j.$$

The resulting total revenue to i is then

$$\alpha[\psi V_j + (1-\psi)V_i].$$

If instead all the potential buyers have a valuation lower than V_i the shares are not sold to anyone else.

Define $V_{-i}^{-} \equiv \{V_j \in (0, V_i), \forall j \neq i\}$ the set of vectors of firm's values V_j such that all values are strictly lower than V_i and V_{-i}^{+} its complement. If all the values V_j are lower than V_i, there will be no trading in the second period, if instead at least one V_j

is higher than V_i, then there will be trading. Then

$$\begin{aligned} U_i(V_i, \tilde{V}_i) \equiv & \int_{V_{-i}^-} \left[\alpha V_i p_i(\tilde{V}_i, V_{-i}) - t_i(\tilde{V}_i, V_{-i})\right] g_{-i}(V_{-i}) dV_{-i} + \\ & + \int_{V_{-i}^+} \left\{\alpha[\psi V_j^{max} + (1-\psi)V_i] p_i(\tilde{V}_i, V_{-i}) - t_i(\tilde{V}_i, V_{-i})\right\} g_{-i}(V_{-i}) dV_{-i}. \end{aligned} \tag{16}$$

where V_j^{max} is the highest value in the vector V_{-i}^+. Appendix A.2 shows that, once again, the first order conditions of the incentive compatibility constraint can be used to transform the objective function of the creditors, as in (8) above, into the following expression.

$$\begin{aligned} & \int_0^{\bar{V}} \left\{ \int_{V_{-i}^-} \sum_i \left[V_i - \alpha \frac{1-F(V_i)}{f(V_i)}\right] p_i(V) dG_{-i}(V_{-i}) \; + \right. \\ & \left. + \int_{V_{-i}^+} \sum_i \left[V_i + \psi\alpha(V_j^{max} - V_i) - (1-\psi)\alpha \frac{1-F(V_i)}{f(V_i)}\right] p_i(V) dG_{-i}(V_{-i}) \right\} dF(V_i) \end{aligned} \tag{17}$$

The intuition behind this expression is quite simple and it is the same one that applies in the case of perfect information: even when the willingness of a bidder is affected by the option to resale, a higher V_i allows the buyer to extract a higher payment, in proportion $1-\psi$, while only a fraction ψ of the highest value is extracted. We now have all the elements to prove that auctioning off the minimum stake that transfers control $\underline{\alpha}$ is optimal.

Proposition 2. *If $F(V)$ has a monotonic increasing hazard rate, the optimal selling procedure when bidders can trade their shares of the company after these shares are allocated is an auction where the creditors sell $\underline{\alpha}$ shares to the highest bidder.*

Proof: Since $F(V)$ has an increasing hazard rate, it is optimal to set $p_i(V) = 1$ for $V_i = V_j^{max}$. Then, the objective function in (17) is monotonic decreasing in α_i. It is therefore optimal to minimize α_i. Moreover, a constant $\alpha_i(V) = \underline{\alpha}$ satisfies the second order conditions of the incentive compatibility constraint as in the case of Proposition 1. ■

The intuition of what is happening is quite clear once we realize the optimal selling mechanism is an auction: the creditors are still selling the control to the buyer with the highest valuation (V_j^{max}), but the payment is determined by the second highest willingness to pay. However, only the fraction of $1 - \psi$ which is extracted is relevant for the payment, and that fraction is decreasing in α.

Notice that also in this case it is optimal to impose a reservation price and not to serve a buyer with valuation $V_i < V^*$ (where V^* is defined as in the previous case), therefore the same analysis applies.

5. Private Benefits from Control

This section analyzes an environment in which the potential buyers of the firm derive private benefits from control. In this case we need to distinguish between the transferable or public benefits (the market value) that the firm produces when in the hand of bidder i, V_i, and the additional non-transferable or private benefits B_i that accrue only to bidder i from controlling the firm. The firm in the hands of different potential buyers produces different public benefits as well as different private benefits.

In this setting it might still be optimal for the creditors not to sell the entire firm. However this result critically depends on whether the public and the private benefits are positive or negatively correlated among the bidders. When they are positively correlated there is no trade-off between the two types of benefits, while when they are negatively correlated there is a trade-off and the result will depend on how acute it is.

Once again in presenting our result we draw a distinction between the analysis of the case in which both private and public benefits are perfectly known and the case in which private and public benefits are privately known.

5.1. The Perfect Information Case with Private Benefits

Positive Correlation. Consider the case in which there is perfect information on the public and private benefits of the two potential buyers for the firm. Further,

assume that the public benefits V_1 and V_2 are positively correlated with the private benefits B_1 and B_2:

$$V_1 < V_2 \qquad \text{and} \qquad B_1 < B_2. \tag{18}$$

A buyer who is more efficient at maximizing the public value of the company is also more able to extract private benefits from control. In this case, if the entire firm is auctioned off, buyer 2 wins and pays

$$V_1 + B_1 \tag{19}$$

Suppose, instead, that only the control stake $\underline{\alpha}$ is auctioned off. The equilibrium price of the auction of $\underline{\alpha}$ shares is: $[\underline{\alpha}V_1 + B_1]$. Indeed this is the maximum willingness to pay of buyer 1 for the control stake of the firm. The total revenue accruing to the creditors is therefore:

$$\underline{\alpha}V_1 + (1 - \underline{\alpha})V_2 + B_1 \tag{20}$$

Clearly the revenues in (20) exceed the revenues in (19). It is therefore optimal to auction off the minimum control stake of the firm. When there is positive correlation, there is no potential conflict between public and private benefits, so the only relevant issue is how to extract as much surplus as possible from the winner of the auction and the same effect identified in the absence of private benefits applies.

Negative Correlation. Consider now the case in which the public benefits V_1 and V_2 and the private benefits B_1 and B_2 are negatively correlated:

$$V_1 < V_2 \qquad \text{and} \qquad B_1 > B_2. \tag{21}$$

In this case it is not always a dominated choice for the creditors to sell the entire firm. In particular we can distinguish the following three cases.

Case 1. The first case is characterized by the following inequality:

$$\underline{\alpha}V_2 + B_2 > \underline{\alpha}V_1 + B_1. \tag{22}$$

Although buyer 1 is better than buyer 2 at extracting private benefits, these are not very high and do not play a very important role. Inequality (22) implies that

$$V_2 + B_2 > V_1 + B_1. \tag{23}$$

The total surplus is therefore maximized if buyer 2 obtains the control of the firm.[11] Buyer 2 obtains the control as long as he buys at least the fraction $\underline{\alpha}$ of the company. If the entire firm is auctioned off, the creditors' returns $V_1 + B_1$ are clearly strictly smaller than the creditors' returns if only the minimum control stake of the firm is auctioned off: $\underline{\alpha}V_1 + (1-\underline{\alpha})V_2 + B_1$. Once again, in order to maximize their revenues, creditors should sell only the minimum control stake of the firm.

In this case, although there is a trade-off between public value and private benefits, the private benefits of control are not high enough to make a substantial difference, so the effect identified in the absence of private benefits dominates.

Case 2. The second case is characterized by the following pair of inequalities:

$$\underline{\alpha}V_2 + B_2 < \underline{\alpha}V_1 + B_1 \tag{24}$$

and

$$V_2 + B_2 > V_1 + B_1. \tag{25}$$

In this case the difference in private benefits is quite high. If only $\underline{\alpha}$ is auctioned off, we can see from (24) that the control is not allocated efficiently: buyer 1 obtains it, instead of buyer 2. This also reduces the revenues of the creditors: to see this, notice

[11] Notice that in the presence of private benefits ex post efficiency imply that the sum of the firm value and the private benefits is maximized.

that (24) and (25) imply that there exists a percentage of shares μ, $\underline{\alpha} < \mu < 1$, such that:

$$\mu V_2 + B_2 = \mu V_1 + B_1. \tag{26}$$

In order to maximize their revenues, the creditors should auction off μ shares of the firm, rather than the entire firm or a fraction $\underline{\alpha}$. Indeed, in this case, from (26), bidder 2 will obtain the control stake of the firm. The creditors' returns will then be

$$\mu V_1 + B_1 + (1 - \mu)V_2 = V_2 + B_2$$

which are clearly higher than the creditors' returns if the entire firm is auctioned off, $V_1 + B_1$. It is worth noticing that in this case the creditors extract the entire surplus from the winning bidder by auctioning off a percentage of the shares of the firm that is strictly bigger than the minimum control stake $\underline{\alpha}$ but strictly smaller than 100 %.

Even in the presence of a substantial conflict between private and public benefits from control, it is still optimal to sell as few shares as possible (compatibly with maximizing the value of the company). The only difference is that now μ is the minimum stake possible, since with a lower fraction the creditors would not sell the company to the buyer who is going to maximize the total surplus. As a consequence, if a fraction lower than μ were sold, the firm control would not be allocated in an efficient way and the value of the minority stake left in the creditors' hands would not be maximized.

Case 3. The last case is characterized by the highest difference in private benefits of control, so that the following inequality holds:

$$V_2 + B_2 \leq V_1 + B_1. \tag{27}$$

Condition (27) implies that if the entire firm is auctioned off bidder 1 obtains the

firm and this is the efficient allocation. The creditors' returns in the latter case are:

$$V_2 + B_2. \tag{28}$$

However given that by assumption $V_2 > V_1$ if the creditors decide to auction off a percentage of the shares γ which is sufficient to transfer the control, $\gamma \geq \underline{\alpha}$ but strictly smaller than 100%, $\gamma < 1$, the creditors' returns are

$$\gamma V_2 + (1 - \gamma)V_1 + B_2. \tag{29}$$

The returns in (28) are clearly higher than the returns in (29). In other words this is the only case in our analysis in which it is strictly optimal for the creditors to auction off the entire firm. This is because in this case benefits of control are very high, so that extracting these benefits is the best the creditors can do.

To summarize, the presence of private benefits of controls introduces a trade-off. The public component of the firm value in the hands of potential buyers requires the creditors to reduce the fraction of the equity sold to the minimum necessary to transfer control. However, the presence of private benefits from control may induce the creditors to sell more than this minimum fraction, in order to make sure that the firm is allocated efficiently. The higher are the private benefits of control (relative to the market value of the firm) the higher the fraction of the equity which should be sold.

5.2. The Private Information Case with Private Benefits

We now move to the case in which private as well as public benefits are private information of the N potential buyers. For tractability, we restrict our analysis to the case in which there exists a linear relationship between private benefits from

control and public or transferable values of the company:[12]

$$B_i = \bar{B} + \beta V_i \tag{30}$$

If $\beta > 0$ we are in a case with positive correlation. If instead $\beta < 0$ we have negative correlation. Then a buyer i who obtains $\alpha \geq \underline{\alpha}$ shares has a payoff

$$\alpha V_i + B_i = \bar{B} + (\alpha + \beta) V_i \tag{31}$$

Under this assumption we characterize the optimal mechanism to sell the company. This mechanism specifies also the fraction of shares to be sold.[13] The mechanism design problem is the same as in Section 3.2, with the only difference that now equation (2) becomes

$$U_i(V_i, \tilde{V}_i) \equiv \int_{V_{-i}} \left\{ \bar{B} + [\alpha + \beta] V_i \, p_i(\tilde{V}_i, V_{-i}) - t_i(\tilde{V}_i, V_{-i}) \right\} g_{-i}(V_{-i}) dV_{-i}. \tag{32}$$

Following the same steps as in Appendix A.1, the objective function can therefore be transformed into:

$$\int_V \left\{ \sum_i \left[(1+\beta) V_i - (\alpha + \beta) \frac{1 - F(V_i)}{f(V_i)} \right] p_i(V) \right\} g(V) dV \tag{33}$$

We have now all the elements to prove the following result.

Proposition 3. *Assume $F(V)$ has a monotonic increasing hazard rate. The optimal selling procedure depends on the value of β:*

A) If $-\beta < \underline{\alpha}$, the optimal mechanism is an auction of $\underline{\alpha}$ shares of the company.

[12] This assumption allows us to analyze the problem without addressing the issue of the multi-dimensionality of the adverse selection faced by the creditors in this setting.

[13] Cornelli and Li (1997) show in a different context that the seller (in this case the creditors) could actually do even better by not committing to a given number of shares to be sold, but by making α contingent on the bids.

B) If $-\beta > \underline{\alpha}$, the optimal mechanism is an auction of α shares of the company, where α is equal either to $\underline{\alpha}$ or to the minimum between β and 1.

Proof: In Case (A) the objective function in (33) is monotonic decreasing in α. It is therefore optimal to minimize α. In Case (B) the objective function is monotonic increasing in α provided that $\alpha \leq -\beta$. Therefore it is optimal to choose the highest α compatible with $\alpha \leq -\beta$ if the choice is to allocate the firm to the bidder that announces the highest V_i. Alternatively, it is optimal to choose the lowest $\alpha = \underline{\alpha}$ provided that the choice is to allocate the firm to the bidder that announces the lowest V_i. In either case the second order conditions are satisfied for a constant α. ■

Case (A) covers all cases with positive correlation ($\beta > 0$) and the cases where β is negative but not very high in absolute value. This is the case where there is no trade off between public and private values (positive correlation) or the cases where the trade off is not very acute (Case 1 of the previous section): the presence of private benefits of control does not change the problem in a substantial way and it is still optimal to sell $\underline{\alpha}$ shares.

Conversely, when $-\beta > \underline{\alpha}$ — Case (B) in Proposition 3 — it is still true that creditors want to sell the minimum possible stake, but if they sell only $\underline{\alpha}$ shares they are going to attract the buyer with the lowest public value V_i. If they want to sell to the buyer with the highest V_i they have to sell at least β shares. Depending on the value of β and on the distribution $F(V_i)$ they can opt for either alternative. The intuition is simply that increasing α is costly. Therefore the creditors will do it only if it will enable them to end up with a more efficient buyer that might increase the value of the minority stake they retain.

Notice that also in this case it is optimal (from the point of view of maximizing the creditors revenues) to impose a reservation price. In other words, it is optimal to sell the company (or a fraction α of its equity) to a buyer i only if he has a valuation $V_i \geq V^*$, where V^* is such that

$$(1+\beta)V^* - (\alpha+\beta)\frac{1-F(V^*)}{f(V^*)} = 0.$$

Once again, it is easy to see that the ex post inefficiency, introduced by the reservation price, is reduced when we decrease α.

6. The Suggested Procedure and the Privatization of Bankruptcy

The key to our proposal of how to sell a company in bankruptcy is to leave the creditors the option to sell less than 100% of the shares of the bankrupt company. This objective can be practically implemented in a number of ways.

One way to proceed would be for example to transform the bankrupt firm in a all equity firm. Then allocate the shares of this new firm to the creditors following whatever procedure is most suitable for the creditors.[14] Once this is done the creditors are required to sell α % of their share so as to transfer the control to the buyer with the highest valuation and retain the $(1-\alpha)$ % of their shares. The percentage α can be chosen so as to maximize the creditors proceed in the way described in Sections 3, 4 and 5 above.

Alternatively the same procedure could be implemented by selling in a centralized manner α % of the shares and distributing, following whatever criterion is preferred by the creditors, both the monetary revenues from the sale and the residual percentage $(1-\alpha)$ % of shares to the creditors ex-post. Either way the final result would be identical.

One could argue that there is no need to centralize and discipline the way in which creditors sell their shares. In other words, we could simply transform the company in an all equity firm, allocate all shares of the new company to the creditors (following any chosen priority rule) and then let the creditors, now shareholders, decide what to do with the firm. This would be equivalent to privatizing the bankruptcy procedure: it is only necessary to define clearly the ownership rights of the creditors on the firm and then they optimally decide what to do with it.

[14] In particular the creditors might want to follow absolute priority rule using for example the procedure suggested in Bebchuk (1988) or might decide not to follow absolute priority rule. Notice that the main point of this paper is completely independent of the distribution of shares.

In this section we show that in the privatized procedure there always exists an equilibrium that coincides with the one derived in the previous sections, one in which the optimally chosen control stake of the equity, α, is allocated in the hands of the buyer who maximizes the firm's value. However, in the privatized procedure there exist also other equilibria, which are both ex post and ex ante inefficient. Hence disciplining the way the creditors proceed in allocating the bankrupt firm is a way to select the efficient equilibria of the game.

Assume that each creditor i is allocated s_i shares and that creditors have to decide whether to sell an amount $\overline{s}_i$ of their shares, $\overline{s}_i \leq s_i$. To keep the treatment as simple as possible we restrict attention to the perfect information environment in which there are only two potential buyers, 1 and 2, for the firm and there are no private benefits from control.[15] We also assume that the creditors only decision is whether to sell or not the amount $\overline{s}_i$ of shares. In other words, provided that creditors are willing to tender their amount $\overline{s}_i$ of shares these shares are allocated to the most efficient buyer in the way suggested in Sections 3 and 4 above.

Assume that the decision whether to tender an amount $\overline{s}_i$ of shares is taken by each creditor simultaneously and independently. We denote p the share price paid by buyer 2 and take $V_1/S \leq p < V_2/S$ where $S = \sum_i s_i$. Clearly a creditor can always decide to sell the remaining shares in his hands $(s_i - \overline{s}_i)$ immediately after the control of the company is transferred in the hands of buyer 2 at the share price (V_2/S).

The game we just described has a multiplicity of equilibria. In particular in the case in which $s_i < \underline{\alpha}$ for any $i = 1, \ldots, N$, there always exists an equilibrium in which each creditor tenders zero shares, since he expects the other creditors to tender zero shares as well. In other words, $\overline{s}_i = 0$ for every $i = 1, \ldots, N$, is always an equilibrium of this tendering game. This equilibrium is clearly ex-post inefficient, since the firm has no (or very low) value in the hands of the creditors while it has value V_2 in the hands of buyer 2. It is also ex-ante inefficient, since the creditors revenues are not maximized. The problem is the coordination failure among the creditors.[16]

[15] The discussion can be easily extended to the case in which there are private benefits from control.

[16] The logic is exactly the same of Grossman and Hart (1980) and Shleifer and Vishny (1986).

It should be noticed, however, that there also exists an equilibrium which reproduces exactly the allocation of shares that we described in Sections 3 and 4 above as the outcome of our suggested procedure. Indeed if creditor i believes that the other creditors will sell exactly the percentage of shares $(\underline{\alpha} - \pi)$ %, where $\pi \leq (s_i/S)$, then creditor i feels pivotal. It is therefore a best reply for creditor i to tender an amount of shares πS. The result is that the control is transferred to buyer 2, the firm value is V_2 and the total revenue obtained by the creditors is $[\underline{\alpha} pS + (1 - \underline{\alpha})V_2]$. This equilibrium is equivalent to the one derived in Section 3 above with a centralized mechanism. Indeed, in the event that $p = (V_1/S)$ the creditors' revenue coincides with the one in (1).

Disciplining and centralizing the procedure the creditors are supposed to use solves the creditors' coordination problem. In other words it isolates as the unique outcome the one which achieves ex-post as well as ex-ante efficiency. It is possible to re-interpret this discussion in favour of bankruptcy procedure that disciplines the way creditors behave in the event of a corporate re-organization.

7. Concluding Remarks

In this paper we propose a way to sell a company in bankruptcy that maximizes the creditor's proceeds. For this purpose creditors should be free to separate the voting rights of the firm from the cash flow rights. In particular in the absence of private benefits from control they should auction off the majority of the voting rights retaining as much as possible of the cash flow rights. This can be done by both selling a low fraction of shares or by changing the voting structure of the shares. When private benefits are present it is not any more optimal to separate completely voting and cash flow rights although creditors might still gain by retaining part of the cash flow rights of the company. Therefore, the creditors' incentive to maximize their proceeds may in general lead to a violation of the one-share-one vote principle at the restructuring stage of a bankruptcy (Grossman and Hart 1988).

This way to sell a company in bankruptcy implies an optimal choice of the minimum stake of the company $\underline{\alpha}$ necessary to transfer control. In the absence of private

benefits from control, it is clearly in the creditors' interests to minimize such stake, for example by auctioning off a minimal number of shares (possibly one share) with all the voting rights. However, the (public) value of a firm under the control from a given buyer (i.e. the expected cash flows when that buyer is in control) may depend on the fraction of cash flow rights that buyer has. In other words, if that buyer owns too little cash flow rights in that company, he may not invest any effort in it and not maximize its value. As a result, the choice of the number of shares with voting rights would not be so extreme (one share would not be optimal). We do not model directly this issue, since it is not crucial for our analysis: one may define $\underline{\alpha}$ as the fraction of the cash flow rights which maximizes that trade-off, and our analysis would then apply with $\underline{\alpha}$ defined in this way.

The presence of private benefits from control may also provide an incentive not to sell the minimum number of shares. In fact, if the private benefits of control are larger the larger is the control stake the buyer obtains (as in Burkart, Gromb, and Panunzi (1998)), the creditors may want to increase the number of shares sold. Once again, our analysis could be extended to consider $\underline{\alpha}$ not as exogenous but as the fraction that maximizes the surplus to be extracted. Our result will still go through once we redefine $\underline{\alpha}$ in that way.

Appendix

A.1. Derivation of the first and second order condition.

The incentive compatibility constraint (5) can be expressed as $V_i = \arg\max_{\tilde{V}_i} U_i(V_i, \tilde{V}_i)$. Assuming differentiability, by envelope theorem

$$\frac{dU_i}{dV_i}(V_i, V_i) = \int_{V_{-i}} \alpha p_i(V_i, V_{-i}) g_{-i}(V_{-i}) dV_{-i}. \tag{A.1}$$

Re-integrating it, we get:

$$U_i(V_i, V_i) = \int_0^{V_i} \int_{V_{-i}} \alpha p_i(x, V_{-i}) g_{-i}(V_{-i}) dV_{-i} dx + U_i(0, 0). \tag{A.2}$$

Comparing the expression for $U_i(V_i, V_i)$ in (A.2) and its definition in (2), solving for t_i, we obtain:

$$\begin{aligned}\int_V t_i(V)g(V)dV = \int_V \alpha V_i p_i(V)g(V)dV - U_i(0,0)+\\ -\int_{V_{-i}} g_{-i}(V_{-i}) \int_0^{\bar{V}} g_i(V_i) \int_0^{V_i} \alpha p_i(x, V_{-i})dx dV_i dV_{-i}.\end{aligned} \tag{A.3}$$

Integrating by parts, the above expression can be transformed into:

$$\int_V t_i(V)g(V)dv = \int_V \alpha \left[V_i - \frac{1-F_i(V_i)}{f_i(V_i)}\right] p_i(V)g(V)dV - U_i(0,0). \tag{A.4}$$

Substituting (A.4) into (3) we obtain equation (8).

The second order condition for the maximization is: $\frac{\partial^2 U_i(V_i,\tilde{V}_i)}{\partial \tilde{V_i}^2} \mid_{\tilde{V}_i=V_i} \leq 0$. Recall the first order condition: $\frac{\partial U_i(V_i,\tilde{V}_i)}{\partial \tilde{V}_i} \mid_{\tilde{V}_i=V_i} \equiv 0$. Differentiating this first order condition on both sides with respect to $\tilde{V}_i$, we have

$$\frac{\partial^2 U_i(V_i,\tilde{V}_i)}{\partial V_i \partial \tilde{V}_i} \mid_{\tilde{V}_i=V_i} + \frac{\partial^2 U_i(V_i,\tilde{V}_i)}{\partial \tilde{V_i}^2} \mid_{\tilde{V}_i=V_i} = 0.$$

Therefore, the second order condition is satisfied if: $\frac{\partial^2 U_i(V_i,\tilde{V}_i)}{\partial V_i \partial \tilde{V}_i} \mid_{\tilde{V}_i=V_i} \geq 0$, which can be rewritten as

$$\int_{V_{-i}} \alpha \frac{\partial p_i(V)}{\partial V_i} g_{-i}(V_{-i})dV_{-i} \geq 0, \quad \forall i \in N,\ \forall V_i \in [0,\bar{V}] \tag{A.5}$$

A.2. Derivation of the first and second order condition with trading

Proceeding as in the case before, by envelope theorem

$$\begin{aligned}\frac{dU_i}{dV_i}(V_i,V_i) = \int_{V_{-i}^-} \alpha p_i(V_i, V_{-i}) g_{-i}(V_{-i})dV_{-i} +\\ + (1-\psi)\int_{V_{-i}^+} \alpha p_i(V_i, V_{-i}) g_{-i}(V_{-i})dV_{-i}.\end{aligned} \tag{A.6}$$

(the effects of a change of V_i on the extremes of integration compensate each other). Re-integrating it, we get:

$$U_i(V_i, V_i) = U_i(0,0) + + \int_0^{V_i} \Big\{ \int_{V_{-i}^-} \alpha p_i(x, V_{-i}) g_{-i}(V_{-i}) dV_{-i} + \\ + (1-\psi) \int_{V_{-i}^+} \alpha p_i(x, V_{-i}) g_{-i}(V_{-i}) dV_{-i} \Big\} dx. \tag{A.7}$$

We can set $U_i(0,0) = 0$ using the individual rationality constraint. Then, comparing the expression for $U_i(V_i, V_i)$ in (A.7) and its definition in (16), solving for t_i, we obtain:

$$\begin{aligned} \int_V t_i(V) g(V) dV = & \int_0^{\bar{V}} \Big\{ \int_{V_{-i}^-} \alpha V_i p_i(V_{-i}) g(V_{-i}) dV_{-i} + \\ & + \int_{V_{-i}^+} \alpha \left[(1-\psi) V_i + \psi V_j^{max} \right] p_i(V_{-i}) g(V_{-i}) dV_{-i} \Big\} f(V_i) dV_i + \\ - \int_0^{\bar{V}} & \Big\{ \int_{V_{-i}^-} \int_0^{V_i} \alpha p_i(x, V_{-i}) g_{-i}(V_{-i}) dx dV_{-i} + \\ & - \int_{V_{-i}^+} \int_0^{V_i} \alpha (1-\psi) p_i(x, V_{-i}) g_{-i}(V_{-i}) dV_{-i} \Big\} f(V_i) dV_i. \end{aligned} \tag{A.8}$$

Integrating by parts, the above expression can be transformed into:

$$\begin{aligned} \int_V t_i(V) g(V) dV = & \int_0^{\bar{V}} \Big\{ \int_{V_{-i}^-} \alpha \left[V_i - \frac{1-F(V_i)}{f(V_i)} \right] p_i(V) dG_{-i}(V_{-i}) + \\ + \alpha \int_{V_{-i}^+} & \left[(1-\psi) \left[V_i - \frac{1-F(V_i)}{f(V_i)} \right] + \psi V_j^{max} \right] p_i(V) dG_{-i}(V_{-i}) \Big\} dF(V_i). \end{aligned} \tag{A.9}$$

Substituting (A.9) into (3) we obtain equation (17).

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